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Optimal Maintenance Scheduling of a Gas Engine Power Plant using Generalized Disjunctive Programming

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A new continuous-time model for long-term scheduling of a gas engine power plant with parallel units is presented. Gas engines are shut down according to a regular maintenance plan that limits the number of hours spent online. To minimize salary expenditure with skilled labor, a single maintenance team is considered which is unavailable during certain periods of time. Other challenging constraints involve constant minimum and variable maximum power demands. The objective is to maximize the revenue from electricity sales assuming seasonal variations in electricity pricing by reducing idle times and shutdowns in high-tariff periods. By first developing a generalized disjunctive programming model and then applying both big-M and hull reformulation techniques, we reduce the burden of finding the appropriate set of mixed-integer linear constraints. Through the solution of a real-life problem, we show that the proposed formulations are very efficient computationally, while gaining valuable insights about the system. *AIChE J*, 60: 2083–2097, 2014
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Introduction

Power systems are becoming increasingly important to chemical engineering. In the context of process design and operation, recent works have been looking into systems subject to: (1) varying power demand[1–3]; (2) fluctuating electricity pricing.[4–10] Concerning the latter, different scheduling models have been proposed to optimize cement,[4] air separation,[5] steel,[6–9] and combined heat and power plants,[10] and important economic benefits have been reported in cases of low capacity utilization. In this work, we study the opportunities arising from seasonal variations in electricity pricing when considering the long-term maintenance scheduling of a power plant providing electricity to a chemical complex.

Industrial sites require regular maintenance to ensure reliability of their equipment and avoid emergency shutdowns. The main concerns of maintenance scheduling are to guarantee feasible material and utility balances while minimizing cost of labor.[11] As in-house skilled labor is limited and external labor is expensive, the maintenance of plants is scheduled so as to make effective use of in-house labor.

The maintenance scheduling of generators in power systems is one of the most significant problems in power systems operation and management.[12] To avoid premature aging and failure of generators leading to unplanned and costly power outages, it is important to carry out preventive maintenance at regular intervals.[13] The maintenance schedule affects many short- and long-term planning functions. For example, unit commitment, fuel scheduling, reliability calculations, and production cost all have a maintenance schedule as input. Therefore, a suboptimal schedule can affect each of these functions adversely.[14]

In centralized power systems, the maintenance scheduling of generator units is usually performed by the system operator and imposed to power plants,[12] but this is no longer valid in currently restructured electric energy systems.[15] The conventional approach for maintenance scheduling now involves interaction between the independent system operator (ISO) and the generation companies (GENCOs). In this process, the objective of the GENCOs is to maximize their annual benefits, favoring unit maintenance in low price weeks.[15] In contrast, the ISO will also try to maximize the reliability of the power grid, seeking a maintenance plan with similar reliability throughout the weeks of the year and preferring maintenance in low demand weeks. Hence, the ISO may return some maintenance requests for modification. To achieve a maintenance plan that meets the target of both producers and operators, Conejo et al.[15] proposed a coordinating mechanism based on incentives/disincentives, where all producers

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are considered to be price takers, that is, market prices are independent of their maintenance schedules. If this condition does not hold, a representation of the market clearing process must be included, leading to the formulation of a problem with equilibrium constraints.[16] A closely related problem is the one dealing with transmission lines.[17]

The power plant considered in this work, primarily supports a chemical complex having limited interaction with the grid. Focus is, thus, on the problem from the power producer perspective.

The maintenance scheduling of thermal generation units is a large-scale combinatorial optimization problem for which a variety of optimization methods have been applied to solve it: mathematical programming, dynamic programming, genetic algorithms, simulated annealing, tabu search and so forth. The objective function is often quadratic[13,14,18] based on economic cost or reliability. Mathematical programming formulations involve a discrete-time representation[12–17] that uses binary variables to identify the time interval in which maintenance starts or is being performed. The time horizon ranges from 1[12–17] to 5 years[18] with the time grid consisting of multiple periods, which can either be uniform in length (e.g., 1 week intervals), or nonuniform, if one wants to take advantage of the differences in electricity pricing within subperiods of a week (e.g., peak, shoulder, and valley for weekdays and weekend).[15]

Four sets of linear constraints need to be enforced in the maintenance scheduling problem: (1) continuity of maintenance activities, where each unit is maintained for a specified length of time without interruption; (2) maintenance window constraints, which define the possible times for execution of the maintenance activities; (3) maximum and minimum power output constraints, which consider the demand and minimum reserve margins of the power system; (4) crew constraints, which consider the manpower availability for maintenance work; the maintenance resource is either the total number of skilled workers available,[12,13] or constraints that specify a maximum number of units that can be maintained at a given time,[14] or that no two units can be maintained simultaneously by the same crew.[18]

Overall, state of the art approaches assume that maintenance for a particular unit occurs just once in the given time horizon and that the maintenance time windows are not affected by the previous operation regime. In the real-life problem addressed in this article, the maintenance of the power plant involves multiple shutdowns for each generator. Maintenance is enforced between a minimum and a maximum number of run hours after the previous shutdown, and so the maintenance time windows are dynamic rather than static or, in other words, they are an endogenous part of the scheduling problem. Generators feature an idle mode besides the online and shutdown modes to save online hours for periods where the electricity price is higher, leading to higher revenue from electricity sales. Although this is straightforward to model with a discrete-time representation, it presents a challenge for a continuous-time model, which is required since the shortest duration of a planned maintenance is just 12 h, a very small value compared to a time horizon of a few years. A similar challenge is associated to time-dependent resource availability constraints, which are quite common in practice (e.g., cost for manpower higher on Sundays).

The novelty of the article is, thus, related to the generation of practical mixed-integer linear constraints for continuous-time models dealing with time/cost-dependent resource availability and shared resources (e.g., a single maintenance team). The solution approach is to start from simpler generalized disjunctive programming (GDP) constraints,[19,20] along the lines of recent work by Castro and Grossmann,[21] who have applied this approach to the key scheduling concepts of immediate, general precedence, and multiple time grids.

Problem Context

The SASOL chemical complex in Sasolburg is reliant on two coal-based power plants to supply steam and electricity to the operating facilities. Growth and fuels compliance projects have resulted in increased demand of steam and electricity. Expansion of the coal-based plants to meet the growth requirements was undesirable due to economic and environmental considerations and so was the alternative of increased reliance on the state owned electricity provider ESKOM for the electricity needs. The new natural gas-based gas engine power plant generates base load electricity for the chemical complex, with excess production being supplied into the national grid. This new plant enables SASOL to reduce operating costs in addition to reducing the carbon footprint in the area.

Electricity generation in the natural gas power plant is varied by stopping or starting engines and not by varying the load per engine as is the case of a gas turbine. An engine runs on a constant base load and is stopped or started as required. It can be taken from offline to 100% load in less than 5 min and stopped in less than 2 min. Thus, the marginal operating cost per engine does not vary and the losses when changing between operation regimes are minimal compared to the running costs. The 18 engines available supply the required flexibility to operate at variable generation loads. Electricity demand first and then the electricity tariff for supplying into the grid, drives the decision of how many engines should be online, while considering that engines accumulate running hours, bringing them closer to their scheduled maintenance shutdowns. The natural gas price remains constant through time and so the marginal cost of generation does not vary on an hourly basis.

Generic Problem Statement

We consider a gas engine power plant producing electricity from natural gas. Each engine $m \in M$ has a given power output pw_m (MW) and at any one time can either be online, in standby, or shutdown mode. To ensure reliability over the years, different types of preventive maintenance shutdowns need to be performed, ranging in duration from half a day to more than 2 weeks. Let $t \in T$ represent an operation time period ending in a shutdown of fixed length $sd_{t,m}$ (h). Preventive shutdowns are recommended after a certain number of online hours (not considering the hours spent in standby mode). To allow for some flexibility on the shutdown schedule, see Figure 1, we assume that online processing times are allowed to vary between given lower $p_{t,m}^L$ and upper bounds $p_{t,m}^U$ (h). Note that the difference $p_{t,m}^U - p_{t,m}^L$ is typically larger for the period t with the longest shutdown.

It is assumed that there is a single maintenance team doing shutdowns, meaning that at most one engine can be on shutdown mode at any time, see Figure 1. To reduce the costs, it may also be convenient to make the maintenance team unavailable in certain periods of time (e.g., on Sundays, Christmas season). For each such period tu, the starting

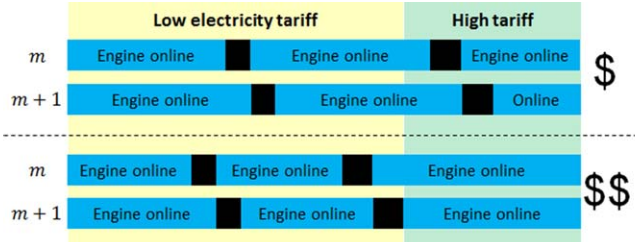

Figure 1. Illustration of optimization goal.

From top to bottom, reducing the engine online hours in periods with low electricity tariff anticipates maintenance shutdowns (black boxes), preventing them to occur in periods with high tariff and thus increasing revenue from electricity sales. Note that engines cannot maintain simultaneously due to the single maintenance crew. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

u^L_{tu} and ending time u^U_{tu} (h) with respect to the origin of the time horizon must be given.

The amount of time spent on standby mode can be varied to decrease production in periods of low power demand, stagger the shutdowns, and take advantage of higher electricity tariffs. For this particular case study, electricity sales in the winter months are at a higher tariff than the sales in summer. Let tp represent a time period of constant electricity price ce_{tp} (\$/MWh) and cp^L_{tp} and cp^U_{tp} (h) its starting and ending time, respectively.

Most of the time, all energy produced can be sold to the market (unlimited demand), but there can be periods up to a few weeks where the maximum power demand pw^U_{td} is limiting. The starting and ending times of time period td are given, respectively, by d^L_{td} and d^U_{td} (h).

The objective is to maximize the revenue from electricity sales for a given number of operation periods $|T|$, subject to a constant minimum power demand, pw^L (MW), and the aforementioned constraints.

Selection of Time Representation Concepts for Scheduling Model

In a recent review article dealing with production scheduling models for industrial applications, Harjunkoski et al.[22] have identified the production environment and the modeling of time as the two most important features of a mathematical programming scheduling formulation. In terms of the production environment, the maintenance scheduling problem addressed here can be viewed as a sequential, single-stage multiproduct continuous plant with parallel units. The specified operation time periods T can be viewed as multiple products, which are scheduled following a predefined sequence with variable processing times and changeovers (the mandatory shutdowns). We also have a maintenance team, a resource that needs to be shared by the different engines, that is not always available, while accounting for time-dependent pricing and demand for electricity.

In view of the given processing characteristics and constraints, deciding on either a discrete or continuous-time approach is not trivial. On the one hand, volatile prices, maximum electricity demand and maintenance team availability, the shared resource, and the minimum power demand would favor discrete time. Conversely, the simple plant topology, the variable processing times, and the fact that the changeover times can be two orders of magnitude smaller than the processing times, would favor continuous time. In the end, the latter feature was decisive since a very large number of time slots would be required by the discrete-time formulation to handle the problem data accurately (preliminary tests have shown the scheduling problem to be intractable even when considering just a few engines). In contrast, the shared resource and the time-dependent profiles, which do not change frequently, can still be handled by a continuous-time formulation despite the use of a more inefficient set of constraints. The constant minimum power demand will be enforced by eliminating the standby mode from as many engines as those required to achieve the minimum demand, plus one.

Continuous-time models can be of different types.[22] Single time grid is preferred for shared resources[4] but is highly inefficient for single-stage plants[23] when compared to multiple time grid models. In this case, it is not required to keep track of the availability of the maintenance team over time, just to make sure that the maintenance tasks do not overlap. General precedence models are known to be capable of modeling this constraint very efficiently and can be extended to multiple discrete resources.[24–26] The continuous-time model to be presented next is hybrid[26,27] in the sense that it relies on two different concepts for time representation: (1) multiple time grids, one per gas engine, to keep track of the execution of the power production tasks; (2) sequencing variables, to handle the single maintenance team constraint. The model has also to account for events occurring at discrete points in time, which define changes in the electricity tariff, power demand, and availability of the maintenance team.

Generalized Disjunctive Programming Formulation

In this section, we highlight the main elements of the scheduling formulation while providing the model constraints in their simplest form. GDP[19,20] is used for this purpose, allowing us to focus on the linear constraints that are associated to each of the decisions, defined by Boolean variables. In the next section, we discuss the transformation of the GDP into mixed-integer linear programming (MILP) formulations using big-M and convex hull reformulations.[28,29]

Timing production and maintenance tasks on gas units

To ensure the minimum power supply constraint, the gas engines M are divided into engines that are always on (either on online or shutdown mode) M^{ON}, and those that can be idle. The latter provide the necessary flexibility to maximize electricity production in periods of higher price.

We use the concept of multiple time grids[23] to determine the timing of the production and maintenance tasks. Each engine features exactly one production and one maintenance task per shutdown period $t \in T$, that is, the given shutdown periods correspond to the slots of every time grid $m \in M$, see Figures 2 and 3. Given that the online time of engine m in slot t is not fixed but allowed to vary in $[p^L_{t,m}, p^U_{t,m}]$, we need to define nonnegative continuous variables $P_{t,m}$. The other nonnegative continuous variables are $Ts_{t,m}$ and $Te_{t,m}$, which identify the starting and ending time of production in slot t of engine m and $Tm_{t,m}$ the beginning of the shutdown period t of engine m.

For engines that can be idle, we need to decide how many times operation can be interrupted before execution of the

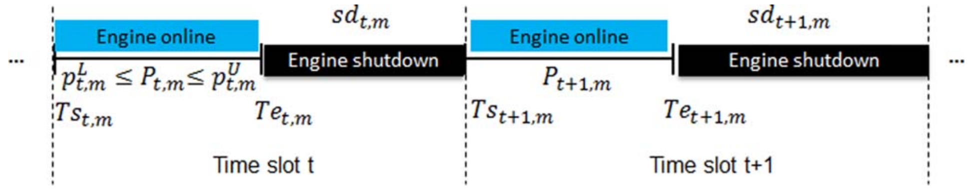

Figure 2. Events occurring within the time slots of always-on gas engines.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

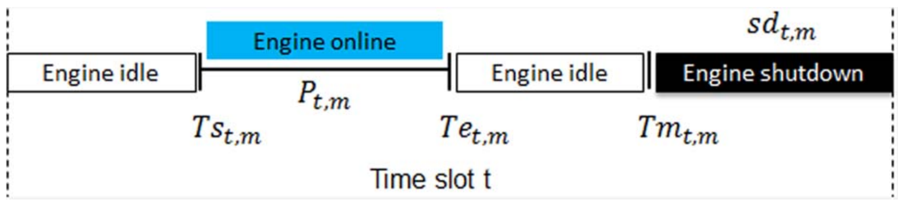

Figure 3. Events occurring within the time slots of gas engines that can be idle.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

shutdown. An operation regime with many interruptions involves high maintenance cost and so it is not desired. Conversely, a very low number may remove the optimal solution from the feasible space. We started to postulate a single occurrence of the idle mode (located before the online mode) only to find out that there were periods of high electricity tariff with engines in standby mode. These disappeared for two occurrences, leading to higher revenue. It was thus assumed a maximum of two occurrences of the idle mode per slot (see Figure 3).

From Figures 2 and 3, the timing constraints are straightforward. Equation 1 states that the ending time is equal to the starting plus processing time. The beginning of the maintenance period coincides with the end of processing for always-on engines, Eq. 2, or is greater or equal for the others, Eq. 3. Then, the starting time of the online mode in slot $t+1$ is either equal to (Eq. 4) or greater or equal than the starting time of the maintenance mode in slot t plus the fixed shutdown length in t (Eq. 5). The starting time of the first slot for always-on units must also be equal to zero, Eq. 6

$$Te_{t,m}=Ts_{t,m}+P_{t,m} \;\; \forall \; t,m \tag{1}$$

$$Tm_{t,m}=Te_{t,m} \;\; \forall \; t,m \in M^{ON} \tag{2}$$

$$Tm_{t,m} \geq Te_{t,m} \;\; \forall \; t,m \in M \backslash M^{ON} \tag{3}$$

$$Ts_{t+1,m}=Tm_{t,m}+sd_{t,m} \;\; \forall \; t,m \in M^{ON} \tag{4}$$

$$Ts_{t+1,m} \geq Tm_{t,m}+sd_{t,m} \;\; \forall \; t,m \in M \backslash M^{ON} \tag{5}$$

$$Ts_{1,m}=0 \;\; \forall \; m \in M^{ON} \tag{6}$$

Depending on the problem data, it is possible to reduce the domain of the model variables and improve computational performance. To facilitate interpretation of the constraints, the variable bounds are written in lower case featuring the same characters as the corresponding model variable, and an extra superscript identifying if it is a lower (L) or an upper (U) bound. As an example $ts^L_{t,m}$ represents the lower bound on the starting time of processing task (t, m), while $tm^U_{t,m}$ indicates the upper bound on the starting time of the shutdown period t of engine m. Equations 7–10, thus define the upper and lower bounds for the timing variables

$$p^L_{t,m} \leq P_{t,m} \leq p^U_{t,m} \;\; \forall \; t,m \tag{7}$$

$$ts^L_{t,m} \leq Ts_{t,m} \leq ts^U_{t,m} \;\; \forall \; t,m \tag{8}$$

$$ts^L_{t,m}+p^L_{t,m}=te^L_{t,m} \leq Te_{t,m} \leq te^U_{t,m}=ts^U_{t,m}+p^U_{t,m} \;\; \forall \; t,m \tag{9}$$

$$tm^L_{t,m} \leq Tm_{t,m} \leq tm^U_{t,m} \;\; \forall \; t,m \tag{10}$$

Sequencing maintenance tasks performed by the single maintenance crew

Enforcing the single maintenance team constraint can be done through the use of general precedence sequencing variables.[30,31] If one considers any pair of tasks (t, m) and (t', m'), there are only two possibilities (hence the use of the exclusive OR in Figure 4), either (t, m) before or after (t', m'). Naturally, there can be other shutdown tasks between the pair being considered. Notice also that there is no need to consider $m=m'$ since Eqs. 1–5 ensure that there is no overlap of shutdown tasks belonging to different slots of the same unit.

Let $Y_{t,m,t',m'}$ be Boolean variables indicating if shutdown of slot t in unit m starts before the shutdown of slot t' in unit m'. The corresponding constraint in Disjunctive Programming form is given by Eq. 11

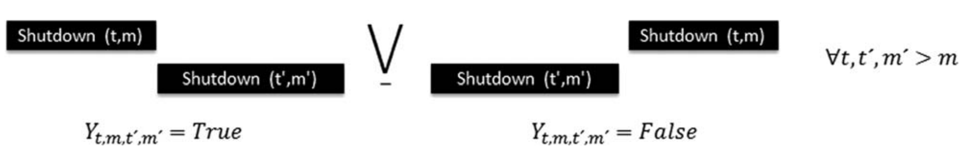

Figure 4. Sequencing pairs of maintenance tasks through general precedence variables.

$$\begin{bmatrix} Y_{t,m,t',m'} \\ Tm_{t,m} + sd_{t,m} \leq Tm_{t',m'} \end{bmatrix} \underline{\bigvee} \begin{bmatrix} \neg Y_{t,m,t',m'} \\ Tm_{t',m'} + sd_{t',m'} \leq Tm_{t,m} \end{bmatrix} \forall\, t, t', m' > m \quad (11)$$

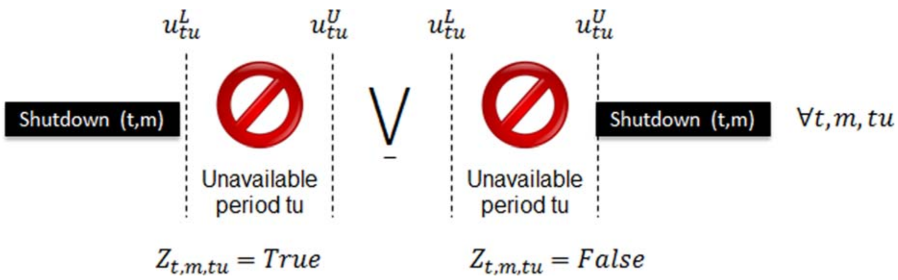

Figure 5. Shutdown tasks cannot take place over unavailable periods of the maintenance team.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Unavailability of the maintenance team in certain periods of time

Another constraint is that the maintenance team it is not available in certain periods of time. Thus, shutdown task (t, m) either ends before the start of unavailable period tu or starts after the end of tu, see Figure 5. Defining $Z_{t,m,tu}$ the new set of general precedence Boolean variables, yields Eq. 12

$$\begin{bmatrix} Z_{t,m,tu} \\ Tm_{t,m} + sd_{t,m} \leq u_{tu}^L \end{bmatrix} \underline{\bigvee} \begin{bmatrix} \neg Z_{t,m,tu} \\ Tm_{t,m} \geq u_{tu}^U \end{bmatrix} \forall\, t, m, tu \quad (12)$$

Calculating the revenue over the different electricity price periods

Dealing with the different electricity tariffs and computing the revenue is the most challenging part of the model. We follow the approach by Nolde and Morari,[6] who have identified six different types of interactions between a processing task and a time period of constant electricity price, when addressing the electric load tracking scheduling problem of a steel plant. Constraints were derived using logic relations[20] and reformulated into MILP format using the big-M technique. However, the MILP constraints were not actually shown. Also using big-M constraints, Haït and Artigues[7] proposed a computationally more efficient model for the exact same problem, while Hadera and Harjunkoski[8] applied the concept to a more complex steel plant.

Let the Boolean variables from $A_{t,m,tp}$ to $F_{t,m,tp}$ account for the six possible location types of processing task (t, m) with respect to constant electricity price period tp, see Figure 6. Type A corresponds to the full duration being located inside the price period, that is, both the starting and ending time of the task must be greater than the interval lower bound and lower than its upper bound. In such case, the time factor $\Delta T_{t,m,tp}$ to consider for computing the revenue, is equal to the processing time $P_{t,m}$, see disjunction further to the left in Eq. 13. If the tasks starts before cp_{tp}^L but ends within $tp\,(B_{t,m,tp}=\text{True})$, the time factor is equal to the difference between the ending time of the task and the interval lower bound. The same can be done for the four remaining alternatives. Note that in the last three disjunctions, the calculation of $\Delta T_{t,m,tp}$ involves only parameters, which facilitates the convex hull reformulation

$$\begin{bmatrix} A_{t,m,tp} \\ Ts_{t,m} \geq cp_{tp}^L \\ Ts_{t,m} \leq cp_{tp}^U \\ Te_{t,m} \geq cp_{tp}^L \\ Te_{t,m} \leq cp_{tp}^U \\ \Delta T_{t,m,tp} = P_{t,m} \end{bmatrix} \underline{\bigvee} \begin{bmatrix} B_{t,m,tp} \\ Ts_{t,m} \leq cp_{tp}^L \\ (Ts_{t,m} \leq cp_{tp}^U) \\ Te_{t,m} \geq cp_{tp}^L \\ Te_{t,m} \leq cp_{tp}^U \\ \Delta T_{t,m,tp} = Te_{t,m} - cp_{tp}^L \end{bmatrix} \underline{\bigvee} \begin{bmatrix} C_{t,m,tp} \\ Ts_{t,m} \geq cp_{tp}^L \\ Ts_{t,m} \leq cp_{tp}^U \\ (Te_{t,m} \geq cp_{tp}^L) \\ Te_{t,m} \geq cp_{tp}^U \\ \Delta T_{t,m,tp} = cp_{tp}^U - Ts_{t,m} \end{bmatrix} \underline{\bigvee}$$

$$\underline{\bigvee} \begin{bmatrix} D_{t,m,tp} \\ Ts_{t,m} \leq cp_{tp}^L \\ (Ts_{t,m} \leq cp_{tp}^U) \\ (Te_{t,m} \geq cp_{tp}^L) \\ Te_{t,m} \geq cp_{tp}^U \\ \Delta T_{t,m,tp} = cp_{tp}^U - cp_{tp}^L \end{bmatrix} \underline{\bigvee} \begin{bmatrix} E_{t,m,tp} \\ (Ts_{t,m} \leq cp_{tp}^L) \\ (Ts_{t,m} \leq cp_{tp}^U) \\ Te_{t,m} \leq cp_{tp}^L \\ (Te_{t,m} \leq cp_{tp}^U) \\ \Delta T_{t,m,tp} = 0 \end{bmatrix} \underline{\bigvee} \begin{bmatrix} F_{t,m,tp} \\ (Ts_{t,m} \geq cp_{tp}^L) \\ Ts_{t,m} \geq cp_{tp}^U \\ (Te_{t,m} \geq cp_{tp}^L) \\ (Te_{t,m} \geq cp_{tp}^U) \\ \Delta T_{t,m,tp} = 0 \end{bmatrix} \forall\, t, m, tp \quad (13)$$

In Eq. 13, redundant constraints are inside parenthesis. As an example, if $C_{t,m,tp}$=True, it is not necessary to consider $Te_{t,m} \geq cp_{tp}^L$, since this is ensured by $Ts_{t,m} \geq cp_{tp}^L$ and Eq. 1. Redundant constraints are shown to make it easier to identify those that are shared by different location variables, something that will be explored in the next section. Overall, for every (t, m, tp) there are 14 nonredundant constraints that need to be reformulated.

Implicit in Eq. 13 is the fact that the sum over all time periods of the time factor variables must be lower than the

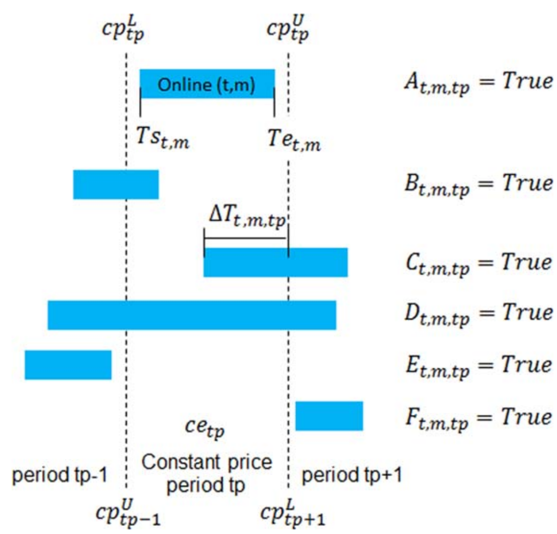

Figure 6. Interaction of processing tasks with periods of constant electricity price periods.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

processing time of the corresponding time period. While not strictly necessarily, Eq. 14 leads to a reduction in the integrality gap and improves computational performance by over one order of magnitude. If time periods tp are sufficiently long to allow execution of all tasks, we can use the equality instead

$$\sum_{tp} \Delta T_{t,m,tp} \leq P_{t,m} \;\; \forall \; t,m \tag{14}$$

To calculate the revenue due to processing task (t,m) in period tp, one just needs to multiply the time factor $\Delta T_{t,m,tp}$, by the electricity price ce_{tp}, and the power output pw_m due to engine m. Thus, the objective function of maximizing the total revenue is given by Eq. 15

$$\max \text{Revenue} = \sum_{t} \sum_{m} \sum_{tp} \Delta T_{t,m,tp} . ce_{tp} . pw_m \tag{15}$$

Ensuring power output does not exceed maximum demand

While in general it is assumed that all energy produced can be sold to the market, it is possible to have times of low power demand during some weeks of the year. Assuming that the maximum power demand $pw_{td}^{\max}$ is higher than the minimum power output $pw^{\min}$ from nonidle engines, we need to account for the other units $(m \in M \backslash M^{\text{ON}})$ that are operating in low demand period td. Operating in period td does not necessarily mean starting and ending within td, that is, any of the first four types of interaction in Figure 6 can occur. Thus, processing task (t,m) can either end before $(X_{t,m,td}^B)$, be active inside or start after $(X_{t,m,td}^A)$ period td, see Figure 7.

The disjunctive programming constraints are given in Eq. 16. Notice that the constraints involving timing variables $Ts_{t,m}$ and $Te_{t,m}$ in disjunction $X_{t,m,td}^{\text{IN}}$ are those shared by types A, B, C, D, while before and after correspond respectively to types E and F

$$\begin{bmatrix} X_{t,m,td}^B \\ Te_{t,m} \leq d_{td}^L \end{bmatrix} \underline{\bigvee} \begin{bmatrix} X_{t,m,td}^{IN} \\ Ts_{t,m} \leq d_{td}^U \\ Te_{t,m} \geq d_{td}^L \end{bmatrix} \underline{\bigvee} \begin{bmatrix} X_{t,m,td}^A \\ Ts_{t,m} \geq d_{td}^U \end{bmatrix} \; \forall \; t, td, m \in M \backslash M^{ON} \tag{16}$$

Equation 17 ensures that the maximum power demand is not exceeded. Note that the second term on the left-hand side gives an upper bound on the power output from always-on engines and not the exact power output since one of the M^{ON} engines may be on maintenance

$$\sum_{t} \sum_{m \in M \backslash M^{\text{ON}}} X_{t,m,td}^{IN} . pw_m + \sum_{m \in M^{\text{ON}}} pw_m \leq pw_{td}^U \;\; \forall \; td \tag{17}$$

Mathematical Programming Formulations

We will be deriving alternative mathematical programming formulations by applying the standard big-M and convex hull reformulations to the GDP formulation presented in the previous section. In the process, all Boolean variables are converted into binary variables, for example, $A_{t,m,tp} = \text{True} \iff A_{t,m,tp} = 1$. To make the linear relaxations as tight as possible, we will be using information from the lower and upper bounds of the model variables (Eqs. 7–10).

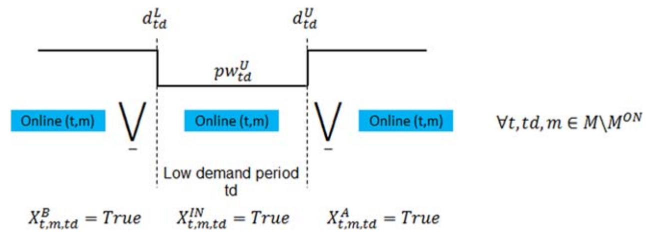

Figure 7. Interaction of processing tasks with low electricity demand periods.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

The disjunctions in Eq. 13 can be reorganized so that a particular timing constraint appears only once. As an example, the condition that the starting time of task (t,m) is greater than lower bound of time period tp is shared by location type A, C, and F while the reverse is true for type B, D, and E. This is reflected in Eq. 18 while the six other possibilities related to variables $Ts_{t,m}$ and $Te_{t,m}$ are part of Eqs. 19–21. Equation 22 deals with the remaining constraints involving the $\Delta T_{t,m,tp}$ variables

$$\begin{bmatrix} A_{t,m,tp} \vee C_{t,m,tp} \vee F_{t,m,tp} \\ Ts_{t,m} \geq cp_{tp}^L \end{bmatrix} \underline{\bigvee} \begin{bmatrix} B_{t,m,tp} \vee D_{t,m,tp} \vee E_{t,m,tp} \\ Ts_{t,m} \leq cp_{tp}^L \end{bmatrix} \forall t, m, tp \tag{18}$$

$$\begin{bmatrix} \neg F_{t,m,tp} \\ Ts_{t,m} \leq cp_{tp}^U \end{bmatrix} \underline{\bigvee} \begin{bmatrix} F_{t,m,tp} \\ Ts_{t,m} \geq cp_{tp}^U \end{bmatrix} \forall t, m, tp \tag{19}$$

$$\begin{bmatrix} E_{t,m,tp} \\ Te_{t,m} \leq cp_{tp}^L \end{bmatrix} \underline{\bigvee} \begin{bmatrix} \neg E_{t,m,tp} \\ Te_{t,m} \geq cp_{tp}^L \end{bmatrix} \forall t, m, tp \tag{20}$$

$$\begin{bmatrix} A_{t,m,tp} \vee B_{t,m,tp} \vee E_{t,m,tp} \\ Te_{t,m} \leq cp_{tp}^U \end{bmatrix} \underline{\bigvee} \begin{bmatrix} C_{t,m,tp} \vee D_{t,m,tp} \vee F_{t,m,tp} \\ Te_{t,m} \geq cp_{tp}^U \end{bmatrix} \forall t, m, tp \tag{21}$$

$$\begin{bmatrix} A_{t,m,tp} \\ \Delta T_{t,m,tp} = P_{t,m} \end{bmatrix} \underline{\bigvee} \begin{bmatrix} B_{t,m,tp} \\ \Delta T_{t,m,tp} = Te_{t,m} - cp_{tp}^L \end{bmatrix} \underline{\bigvee} \begin{bmatrix} C_{t,m,tp} \\ \Delta T_{t,m,tp} = cp_{tp}^U - Ts_{t,m} \end{bmatrix} \underline{\bigvee} \begin{bmatrix} D_{t,m,tp} \\ \Delta T_{t,m,tp} = cp_{tp}^U - cp_{tp}^L \end{bmatrix} \underline{\bigvee} \begin{bmatrix} E_{t,m,tp} \vee F_{t,m,tp} \\ \Delta T_{t,m,tp} = 0 \end{bmatrix} \; \forall \; t, m, tp \tag{22}$$

Overall, there are eight sets of constraints in Eqs. 18–21 as opposed to 14 in Eq. 13. It should also be highlighted that applying basic steps (Trespalacios and Grossmann, submitted for publication)[29] to Eqs. 18–21 would yield Eq. 13 for every (t,m,tp).

As will be seen in the Computational Results section, the advantage of the reorganized disjunctions comes from the generation of fewer constraints for the big-M reformulation, which are also tighter due to the presence of multiple binary variables. In contrast, the quality of the hull relaxation is not as good due to the use of weaker bounds in some of the constraints, and more constraints are actually required. Thus, in the following, we will be deriving the big-M reformulation of Eqs. 18–22 and the hull reformulation of Eq. 13.

Big-M reformulation

We start by reformulating Eq. 11 that avoids maintenance tasks to occur simultaneously. By following the general guidelines[28] to generate the MILP constraints and compute the tightest possible values for the big-M parameters, which were illustrated by Castro and Grossmann[21] in the context of scheduling formulations, Eqs. 23 and 24 can be obtained

$$Tm_{t,m}+sd_{t,m} \leq Tm_{t',m'}+(tm^U_{t,m}-tm^L_{t',m'}+sd_{t,m}) \cdot (1-Y_{t,m,t',m'}) \ \forall \ t,t',m'>m \tag{23}$$

$$Tm_{t',m'}+sd_{t',m'} \leq Tm_{t,m}+(tm^U_{t',m'}-tm^L_{t,m}+sd_{t',m'}) \cdot Y_{t,m,t',m'} \ \forall \ t,t',m'>m \tag{24}$$

Equations 25 and 26 correspond to the reformulation of the two disjunctions in Eq. 12, which ensures that the maintenance team cannot be assigned to work on unavailable periods

$$Tm_{t,m}+sd_{t,m} \leq u^L_{tu}+\left(tm^U_{t,m}-u^L_{tu}+sd_{t,m}\right) \cdot \left(1-Z_{t,m,tu}\right) \ \forall \ t,m,tu \tag{25}$$

$$Tm_{t,m} \geq u^U_{tu}-\left(u^U_{tu}-tm^L_{t,m}\right) \cdot Z_{t,m,tu} \ \forall \ t,m,tu \tag{26}$$

The timing constraints related to the location of the processing tasks with respect to the constant electricity time periods are given by Eqs. 27–34. They correspond to the reformulation of the inequalities inside the disjunctions in Eqs. 18–21. Note that we have used the conversion[20,21] of logic constraints of the type $W_{t,m,tp} \iff A_{t,m,tp} \vee C_{t,m,tp} \vee F_{t,m,tp}$, featuring the model's Boolean variables and the auxiliary variable $W_{t,m,tp}$, into MILP format $W_{t,m,tp}=A_{t,m,tp}+C_{t,m,tp}+F_{t,m,tp}$, with 0–1 variables

$$Ts_{t,m} \geq cp^L_{tp}-\left(cp^L_{tp}-ts^L_{t,m}\right) \cdot \left(1-A_{t,m,tp}-C_{t,m,tp}-F_{t,m,tp}\right) \ \forall \ t,m,tp \tag{27}$$

$$Ts_{t,m} \leq cp^L_{tp}+\left(ts^U_{t,m}-cp^L_{tp}\right) \cdot \left(1-B_{t,m,tp}-D_{t,m,tp}-E_{t,m,tp}\right) \ \forall \ t,m,tp \tag{28}$$

$$Ts_{t,m} \leq cp^U_{tp}+\left(ts^U_{t,m}-cp^U_{tp}\right) \cdot F_{t,m,tp} \ \forall \ t,m,tp \tag{29}$$

$$Ts_{t,m} \geq cp^U_{tp}-\left(cp^U_{tp}-ts^L_{t,m}\right) \cdot \left(1-F_{t,m,tp}\right) \ \forall \ t,m,tp \tag{30}$$

$$Te_{t,m} \leq cp^L_{tp}+\left(te^U_{t,m}-cp^L_{tp}\right) \cdot \left(1-E_{t,m,tp}\right) \ \forall \ t,m,tp \tag{31}$$

$$Te_{t,m} \geq cp^L_{tp}-\left(cp^L_{tp}-te^L_{t,m}\right) \cdot E_{t,m,tp} \ \forall \ t,m,tp \tag{32}$$

$$Te_{t,m} \leq cp^U_{tp}+\left(te^U_{t,m}-cp^U_{tp}\right) \cdot \left(1-A_{t,m,tp}-B_{t,m,tp}-E_{t,m,tp}\right) \ \forall \ t,m,tp \tag{33}$$

$$Te_{t,m} \geq cp^U_{tp}-\left(cp^U_{tp}-te^L_{t,m}\right) \cdot \left(1-C_{t,m,tp}-D_{t,m,tp}-F_{t,m,tp}\right) \ \forall \ t,m,tp \tag{34}$$

It should be emphasized at this point that the equivalent constraints to Eqs. 27–34 in Hadera and Harjunkoski[8] are larger in number and weaker due to the presence of fewer binary variables (except in Eqs. 30 and 31) and the use of a single and hence necessarily larger big-M value in the constraints. The steel problem[8,9] also features fixed rather than variable processing times, allowing for the simplification of Eq. 35. Interestingly, the computation of the equivalent variables to $\Delta T_{t,m,tp}$ resembles Eq. 68 for the hull relaxation, but a different set of variables is used, which appear from the exact linearization of bilinear terms involving continuous and binary variables.

The next set of constraints calculate the time taken by processing task (t,m) in period tp. Because we are dealing with equality constraints, these first need to be divided into two inequality constraints. Taking as example the constraint inside the $A_{t,m,tp}$ disjunction in Eq. 22 $\Delta T_{t,m,tp}=P_{t,m} \iff \Delta T_{t,m,tp} \geq P_{t,m} \wedge \Delta T_{t,m,tp} \leq P_{t,m}$. The values of the big-M parameters are then $\max\left(P_{t,m}-\Delta T_{t,m,tp}\right)=p^U_{t,m}$ and $\max\left(\Delta T_{t,m,tp}-P_{t,m}\right)=0$, leading to Eqs. 35 and 36

$$\Delta T_{t,m,tp} \geq P_{t,m}-p^U_{t,m} \cdot \left(1-A_{t,m,tp}\right) \ \forall \ t,m,tp \tag{35}$$

$$\Delta T_{t,m,tp} \leq P_{t,m} \ \forall \ t,m,tp \tag{36}$$

The remaining constraints are obtained in a similar fashion (Eqs. 37–42). Notice that the global constraints in Eqs. 36 and 42, stating that $\Delta T_{t,m,tp}$ cannot be neither higher than the task's processing time nor than the time interval length, appear naturally from the derivation

$$\Delta T_{t,m,tp} \geq Te_{t,m}-cp^L_{tp}-(te^U_{t,m}-cp^L_{tp}) \cdot \left(1-B_{t,m,tp}\right) \ \forall \ t,m,tp \tag{37}$$

$$\Delta T_{t,m,tp} \leq Te_{t,m}-cp^L_{tp}+(cp^L_{tp}-ts^L_{t,m}) \cdot \left(1-B_{t,m,tp}\right) \ \forall \ t,m,tp \tag{38}$$

$$\Delta T_{t,m,tp} \geq cp^U_{tp}-Ts_{t,m}-(cp^U_{tp}-ts^L_{t,m}) \cdot \left(1-C_{t,m,tp}\right) \ \forall \ t,m,tp \tag{39}$$

$$\Delta T_{t,m,tp} \leq cp^U_{tp}-Ts_{t,m}+(te^U_{t,m}-cp^U_{tp}) \cdot \left(1-C_{t,m,tp}\right) \ \forall \ t,m,tp \tag{40}$$

$$\Delta T_{t,m,tp} \geq (cp^U_{tp}-cp^L_{tp}) \cdot D_{t,m,tp} \ \forall \ t,m,tp \tag{41}$$

$$\Delta T_{t,m,tp} \leq cp^U_{tp}-cp^L_{tp} \ \forall \ t,m,tp \tag{42}$$

$$\Delta T_{t,m,tp} \leq \min\left(cp^U_{tp}-cp^L_{tp}, p^U_{t,m}\right) \cdot \left(1-E_{t,m,tp}-F_{t,m,tp}\right) \ \forall \ t,m,tp \tag{43}$$

Four sets of big-M constraints are required to reformulate Eq. 16. Equation 44 states that if processing task (t,m) is executed before time period td, then the ending time must be lower than the start of the low maximum demand period. Notice that there is no need to use big-M constraints for the starting time variables due to Eq. 1. The same applies when enforcing the starting time variables to be greater than the ending time of period td whenever the processing task is executed after td, see Eq 45. If, conversely, part of the task takes place within td, we need to enforce the bounds in Eqs 46 and 47

$$Te_{t,m} \leq d^L_{td}+\left(te^U_{t,m}-d^L_{td}\right) \cdot \left(1-X^B_{t,m,td}\right) \ \forall \ t,td,m \in M \backslash M^{\mathrm{ON}} \tag{44}$$

$$Ts_{t,m} \geq d_{td}^{U} - \left(d_{td}^{U} - ts_{t,m}^{L}\right) \cdot \left(1 - X_{t,m,td}^{A}\right) \ \forall \ t, td, m \in M \backslash M^{\mathrm{ON}} \tag{45}$$

$$Ts_{t,m} \leq d_{td}^{U} + \left(ts_{t,m}^{U} - d_{td}^{U}\right) \cdot \left(1 - X_{t,m,td}^{IN}\right) \ \forall \ t, td, m \in M \backslash M^{\mathrm{ON}} \tag{46}$$

$$Te_{t,m} \geq d_{td}^{L} - \left(d_{td}^{L} - te_{t,m}^{L}\right) \cdot \left(1 - X_{t,m,td}^{IN}\right) \ \forall \ t, td, m \in M \backslash M^{\mathrm{ON}} \tag{47}$$

Hull reformulation

Compared to the big-M reformulation, the convex hull reformulation involves additional disaggregated variables and constraints. Hence, the benefits in solution time from a stronger linear relaxation may be surpassed by the difficulties resulting from a larger problem size[29] and is often difficult to predict the best performer. The current problem, with its four sets of disjunctive constraints (Eqs. 11–13 and 16) involving different sets of variables, is a good opportunity to gain valuable knowledge concerning the identification of problems where the additional modeling effort required for the derivation of the hull reformulation may be compensated by the improved computational performance.

The first set of constraints (Eq. 11) ensures that the maintenance team is not assigned to two tasks simultaneously and involves timing variables $Tm_{t,m}$ and $Tm_{t',m'}$ for disjunction (t, t', m, m'). The convex hull reformulation for this part of the model requires disaggregated variables with six indices (e.g., $\widehat{Tm}_{t',m',t,m,t',m'}^{Y}$) together with 12 sets of constraints, as can be seen in Appendix A. It compares with the just two sets of constraints required by the big-M reformulation, which is responsible for a significantly smaller mathematical problem and considerably better performance. These results are consistent with those for the single-stage general precedence formulation in Castro and Grossmann.[21]

Fortunately, the results for the other three sets of constraints are more encouraging, as will be seen later on. We start with the unavailability of the maintenance team constraint in Eq. 12, which involves a single variable ($Tm_{t,m}$) and one or two parameters in each disjunction (t, m, tu). Since the indices of the variable are shared with the constraint domain, the new disaggregated variables, $\widehat{Tm}_{t,m,tu}^{Z}$ and $\widehat{Tm}_{t,m,tu}^{\neg Z}$, only feature one additional index compared to the original variable. Furthermore, the single constraint inside the disjunction can be viewed as a bounding constraint, acting as an upper bound on the execution of the maintenance task for the left (Z) disjunction and as a lower bound for the right disjunction ($\neg Z$), see Eqs. 48 and 49. As a consequence, we will be requiring fewer additional constraints than expected. In particular, the fifth set of constraints in Eq. 50 states that the sum of disaggregated variables associated to the two disjunctions, must be equal to the original variable for every unavailable period tu

$$tm_{t,m}^{L} \cdot Z_{t,m,tu} \leq \widehat{Tm}_{t,m,tu}^{Z} \leq (u_{tu}^{L} - sd_{t,m}) \cdot Z_{t,m,tu} \ \forall \ t, m, tu \tag{48}$$

$$u_{tu}^{U} \cdot (1 - Z_{t,m,tu}) \leq \widehat{Tm}_{t,m,tu}^{\neg Z} \leq tm_{t,m}^{U} \cdot (1 - Z_{t,m,tu}) \ \forall \ t, m, tu \tag{49}$$

$$Tm_{t,m} = \widehat{Tm}_{t,m,tu}^{Z} + \widehat{Tm}_{t,m,tu}^{\neg Z} \ \forall \ t, m, tu \tag{50}$$

The majority of the constraints required for calculating the revenue on the different electricity time periods also act as bounding constraints on the timing variables. If one focuses on the constraints involving timing variables $Ts_{t,m}$ and $Te_{t,m}$ for location $A_{t,m,tp}$ in Eq. 13, the hull reformulation is given by Eqs. 51 and 52. It should be highlighted that the min and max functions have the purpose of merging the information from the bounding constraints in Eqs. 8 and 9 with the electricity time period bounds cp_{tp}^{L} and cp_{tp}^{U}

$$\max\left[ts_{t,m}^{L}, cp_{tp}^{L}\right] \cdot A_{t,m,tp} \leq \widehat{Ts}_{t,m,tp}^{A} \leq \min\left[ts_{t,m}^{U}, cp_{tp}^{U}\right] \cdot A_{t,m,tp} \ \forall \ t, m, tp \tag{51}$$

$$\max\left[te_{t,m}^{L}, cp_{tp}^{L}\right] \cdot A_{t,m,tp} \leq \widehat{Te}_{t,m,tp}^{A} \leq \min\left[te_{t,m}^{U}, cp_{tp}^{U}\right] \cdot A_{t,m,tp} \ \forall \ t, m, tp \tag{52}$$

The MILP constraints for the five other location types are given in Eqs. 53–62. Notice that most redundant constraints inside parenthesis in Eq. 13 do not appear. In such cases, the information from the upper/lower bounds comes exclusively from the bounding constraints in Eqs. 8 and 9, avoiding the need for the min/max functions (e.g., check the left-hand side of Eq. 53)

$$ts_{t,m}^{L} \cdot B_{t,m,tp} \leq \widehat{Ts}_{t,m,tp}^{B} \leq \min\left[ts_{t,m}^{U}, cp_{tp}^{L}\right] \cdot B_{t,m,tp} \ \forall \ t, m, tp \tag{53}$$

$$\max\left[te_{t,m}^{L}, cp_{tp}^{L}\right] \cdot B_{t,m,tp} \leq \widehat{Te}_{t,m,tp}^{B} \leq \min\left[te_{t,m}^{U}, cp_{tp}^{U}\right] \cdot B_{t,m,tp} \ \forall \ t, m, tp \tag{54}$$

$$\max\left[ts_{t,m}^{L}, cp_{tp}^{L}\right] \cdot C_{t,m,tp} \leq \widehat{Ts}_{t,m,tp}^{C} \leq \min\left[ts_{t,m}^{U}, cp_{tp}^{U}\right] \cdot C_{t,m,tp} \ \forall \ t, m, tp \tag{55}$$

$$\max\left[te_{t,m}^{L}, cp_{tp}^{U}\right] \cdot C_{t,m,tp} \leq \widehat{Te}_{t,m,tp}^{C} \leq te_{t,m}^{U} \cdot C_{t,m,tp} \ \forall \ t, m, tp \tag{56}$$

$$ts_{t,m}^{L} \cdot D_{t,m,tp} \leq \widehat{Ts}_{t,m,tp}^{D} \leq \min\left[ts_{t,m}^{U}, cp_{tp}^{L}\right] \cdot D_{t,m,tp} \ \forall \ t, m, tp \tag{57}$$

$$\max\left[te_{t,m}^{L}, cp_{tp}^{U}\right] \cdot D_{t,m,tp} \leq \widehat{Te}_{t,m,tp}^{D} \leq te_{t,m}^{U} \cdot D_{t,m,tp} \ \forall \ t, m, tp \tag{58}$$

$$ts_{t,m}^{L} \cdot E_{t,m,tp} \leq \widehat{Ts}_{t,m,tp}^{E} \leq \min\left[ts_{t,m}^{U}, cp_{tp}^{L}\right] \cdot E_{t,m,tp} \ \forall \ t, m, tp \tag{59}$$

$$te_{t,m}^{L} \cdot E_{t,m,tp} \leq \widehat{Te}_{t,m,tp}^{E} \leq \min\left[te_{t,m}^{U}, cp_{tp}^{L}\right] \cdot E_{t,m,tp} \ \forall \ t, m, tp \tag{60}$$

$$\max\left[ts_{t,m}^{L}, cp_{tp}^{U}\right] \cdot F_{t,m,tp} \leq \widehat{Ts}_{t,m,tp}^{F} \leq ts_{t,m}^{U} \cdot F_{t,m,tp} \ \forall \ t, m, tp \tag{61}$$

$$\max\left[te_{t,m}^{L}, cp_{tp}^{U}\right] \cdot F_{t,m,tp} \leq \widehat{Te}_{t,m,tp}^{F} \leq te_{t,m}^{U} \cdot F_{t,m,tp} \ \forall \ t, m, tp \tag{62}$$

Equations 63 and 64 provide the relation between the original and disaggregated variables

$$Ts_{t,m}=\widehat{Ts}^{A}_{t,m,tp}+\widehat{Ts}^{B}_{t,m,tp}+\widehat{Ts}^{C}_{t,m,tp}+\widehat{Ts}^{D}_{t,m,tp}+\widehat{Ts}^{E}_{t,m,tp}+\widehat{Ts}^{F}_{t,m,tp}\ \forall\ t,m,tp \quad (63)$$

$$Te_{t,m}=\widehat{Te}^{A}_{t,m,tp}+\widehat{Te}^{B}_{t,m,tp}+\widehat{Te}^{C}_{t,m,tp}+\widehat{Te}^{D}_{t,m,tp}+\widehat{Te}^{E}_{t,m,tp}+\widehat{Te}^{F}_{t,m,tp}\ \forall\ t,m,tp \quad (64)$$

As for the constraints involving the time factor variables $\Delta T_{t,m,tp}$, it is required to define new disaggregated variables linked to the processing times $P_{t,m}$. However, unlike what we have seen so far, these variables appear in a single disjunction in Eq. 13, so it suffices to define two sets of disaggregated variables, one for the disjunction where they appear ($\widehat{P}^{A}_{t,m,tp}$) and another for the other cases (e.g., $\widehat{P}^{\neg A}_{t,m,tp}$). The required constraints are given in Eqs. 65–67

$$p^{L}_{t,m}\cdot A_{t,m,tp}\le \widehat{P}^{A}_{t,m,tp}\le \min\left(p^{U}_{t,m},cp^{U}_{tp}-cp^{L}_{tp}\right)\cdot A_{t,m,tp}\ \forall\ t,m,tp \quad (65)$$

$$p^{L}_{t,m}\cdot(1-A_{t,m,tp})\le \widehat{P}^{\neg A}_{t,m,tp}\le p^{U}_{t,m}\cdot(1-A_{t,m,tp})\ \forall\ t,m,tp \quad (66)$$

$$P_{t,m}=\widehat{P}^{A}_{t,m,tp}+\widehat{P}^{\neg A}_{t,m,tp}\ \forall\ t,m,tp \quad (67)$$

The time factor $\Delta T_{t,m,tp}$ can then be written as a sum of multiple terms, Eq. 68. The full set of constraints of the hull relaxation of Eqs. 18–22 is given in Appendix B

$$\Delta T_{t,m,tp}=\widehat{P}^{A}_{t,m,tp}+\widehat{Te}^{B}_{t,m,tp}-cp^{L}_{tp}\cdot B_{t,m,tp}+cp^{U}_{tp}\cdot C_{t,m,tp}-\widehat{Ts}^{C}_{t,m,tp}+(cp^{U}_{tp}-cp^{L}_{tp})\cdot D_{t,m,tp}\ \forall\ t,m,tp \quad (68)$$

Finally, we have the constraints identifying the tasks executed in the low electricity demand periods, Eqs. 69–76. Notice the three additional sets of disaggregated starting and ending time variables

$$ts^{L}_{t,m}\cdot X^{B}_{t,m,td}\le \widehat{Ts}^{XB}_{t,m,td}\le \min\left[ts^{U}_{t,m},\left(d^{L}_{td}-p^{L}_{t,m}\right)\right]\cdot X^{B}_{t,m,td}\quad \forall\ t,td,m\in M\backslash M^{ON} \quad (69)$$

$$te^{L}_{t,m}\cdot X^{B}_{t,m,td}\le \widehat{Te}^{XB}_{t,m,td}\le \min\left[te^{U}_{t,m},d^{L}_{td}\right]\cdot X^{B}_{t,m,td}\quad \forall\ t,td,m\in M\backslash M^{ON} \quad (70)$$

$$\max\left[ts^{L}_{t,m},(d^{L}_{td}-p^{U}_{t,m})\right]\cdot X^{IN}_{t,m,td}\le \widehat{Ts}^{XIN}_{t,m,td}\le \min\left[ts^{U}_{t,m},d^{U}_{td}\right]\cdot X^{IN}_{t,m,td}\quad \forall\ t,td,m\in M\backslash M^{ON} \quad (71)$$

$$\max\left[te^{L}_{t,m},d^{L}_{td}\right]\cdot X^{IN}_{t,m,td}\le \widehat{Te}^{XIN}_{t,m,td}\le \min\left[te^{U}_{t,m},(d^{U}_{td}+p^{U}_{t,m})\right]\cdot X^{IN}_{t,m,td}\quad \forall\ t,td,m\in M\backslash M^{ON} \quad (72)$$

$$\max\left[ts^{L}_{t,m},d^{U}_{td}\right]\cdot X^{A}_{t,m,td}\le \widehat{Ts}^{XA}_{t,m,td}\le ts^{U}_{t,m}\cdot X^{A}_{t,m,td}\quad \forall\ t,td,m\in M\backslash M^{ON} \quad (73)$$

$$\max\left[te^{L}_{t,m},(d^{U}_{td}+p^{L}_{t,m})\right]\cdot X^{A}_{t,m,td}\le \widehat{Te}^{XA}_{t,m,td}\le te^{U}_{t,m}\cdot X^{A}_{t,m,td}\quad \forall\ t,td,m\in M\backslash M^{ON} \quad (74)$$

$$Ts_{t,m}=\widehat{Ts}^{XB}_{t,m,td}+\widehat{Ts}^{XIN}_{t,m,td}+\widehat{Ts}^{XA}_{t,m,td}\ \forall\ t,td,m\in M\backslash M^{ON} \quad (75)$$

$$Te_{t,m}=\widehat{Te}^{XB}_{t,m,td}+\widehat{Te}^{XIN}_{t,m,td}+\widehat{Te}^{XA}_{t,m,td}\ \forall\ t,td,m\in M\backslash M^{ON} \quad (76)$$

Common constraints

The big-M and convex hull reformulation share the constraint that exactly one type of interaction of processing task (t,m) with constant electricity price period tp must be selected

$$A_{t,m,tp}+B_{t,m,tp}+C_{t,m,tp}+D_{t,m,tp}+E_{t,m,tp}+F_{t,m,tp}=1\ \forall\ t,m,tp \quad (77)$$

Similarly, processing task (t,m) is either completely before, completely after, or partially within lower power demand period td

$$X^{B}_{t,m,td}+X^{IN}_{t,m,td}+X^{A}_{t,m,td}=1\ \forall\ t,td,m\in M\backslash M^{ON} \quad (78)$$

Case Study

We consider a power plant with 18 identical gas engines with a generation capacity per engine $pw_m=10$ MW $\forall m$. The minimum electricity production should be $pw^L=80$ MW, corresponding to a minimum of $|M^{ON}|=9$ engines operating online simultaneously (recall that this class is either online or in shutdown mode and that at most one engine is in shutdown mode). The schedule involves $|T|=12$ operation time periods, with the number of online hours ranging between [2000, 2500] except for $t=8$, which ends in a major shutdown. The duration of the required maintenance shutdowns is also given in Table 1.

The production plan is to be obtained for roughly 3.5 years, comprising eight time periods of constant electricity price, where the low tariff is intercalated with the high tariff, see Table 2. There are also four periods of maximum power demand lasting 3 weeks each (Table 3) and the maintenance team is unavailable 1 week around Christmas (Table 4).

Most of the complexity of the model arises from the four-index binary variables $Y_{t,m,t',m'}$. Given that the gas engines are identical, we can assume that within operation period t, the shutdown of unit m precedes the shutdown of unit $m'>m$. Then, due to the processing time constraints, it can also be ensured that shutdown (t,m) precedes shutdown (t',m) for $t'>t$. The general condition, given in Eq. 79, is responsible for orders of magnitude reduction in computational time. In later periods, it may also occur that shutdown $(t+1,m)$ of low index units occurs before (t,m') of high-index units, but that is a decision to be determined by the optimization

$$Y_{t,m,t',m'}=1\ \forall\ t'\ge t,m'>m \quad (79)$$

Bounding the model variables

Most constraints need information from the lower and upper bounds of the timing variables. Based on the chosen order for the shutdown tasks, one can derive rigorous lower bounds for the start of the maintenance tasks ($tm^{L}_{t,m}$) based on Figure 8. Conversely, the lower bound for shutdown (t,m) may be the preceding processing task (t,m), which lasts a minimum of $p^{L}_{t,m}$ (e.g., slot 1 for unit 1). Conversely, the limiting factor for unit $m+1$ may be the end of shutdown task (t,m).

The required lower bounds are calculated according to Eq. 80. As for the upper bounds, we use the same formula replacing the lower bounding parameters with their upper bounding counterparts. However, it should be highlighted that upper bounds are heuristic since periods of low power demand and maintenance team unavailability may cause further delays, possibly even compromising feasibility. The reason why this was not observed is probably due to the large

Table 1. Bounds on Time Spent Online and Length of Shutdown Periods (Same for All Engines) (h)

t	1	2	3	4	5	6	7	8	9	10	11	12
$p_{t,m}^U$	2500	2500	2500	2500	2500	2500	2500	3000	2500	2500	2500	2500
$p_{t,m}^L$	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
$sd_{t,m}$	12	72	12	96	12	120	12	432	12	72	12	120

Table 2. Electricity Cost Data

tp	1	2	3	4	5	6	7	8
cp_{tp}^U (h)	3624	5832	12,384	14,592	21,144	23,352	29,928	32,136
cp_{tp}^L (h)	0	3624	5832	12,384	14,592	21,144	23,352	29,928
ce_{tp} (\$/MWh)	40	75	40	75	40	75	40	75

flexibility of online hours before shutdown and the small number of such relatively short periods

FOR $m=1$ *to* $|M|$

FOR $t=1$ *to* $|T|$

$$tm_{t,m}^L=\max\left(tm_{t,m-1}^L+sd_{t,m-1},\, tm_{t-1,m}^L+sd_{t-1,m}+p_{t,m}^L\right)$$

$$tm_{t,m}^U=\max\left(tm_{t,m-1}^U+sd_{t,m-1},\, tm_{t-1,m}^U+sd_{t-1,m}+p_{t,m}^U\right)$$

NEXT t

NEXT m

(80)

The bounds for the starting time variables are given by Eqs. 81 and 82. In Eq. 82, the right-hand side has a conditional domain since the starting time of always-on units in the first slot is equal to 0, recall Eq. 6

$$ts_{t,m}^L=tm_{t-1,m}^L+sd_{t-1,m}\ \forall\ t,m \tag{81}$$

$$ts_{t,m}^U=\max\left[\max\left(0, tm_{t,m}^U-p_{t,m}^U\right),\ tm_{t-1,m}^U+sd_{t-1,m}\right]\Big|_{m\in M\setminus M^{ON}\vee t>1}\ \forall\ t,m \tag{82}$$

Computational Results

The MILP models resulting from the big-M and hull reformulations of the GDP model were implemented in GAMS 24.1 and solved by CPLEX 12.5 using a single thread and default options up to a relative optimality tolerance $=10^{-6}$ or maximum computational time $=3600$ CPUs. The hardware consisted on a desktop with an Intel i7 950 (3.07 GHz) with 8 GB of RAM running Windows 7.

Table 3. Periods of Maximum Power Demand

td	1	2	3	4
d_{td}^U (h)	2664	11,424	20,184	28,968
d_{td}^L (h)	2160	10,920	19,680	28,464
pw_{td}^U (MW)	140	140	140	140

Table 4. Periods of Maintenance Team Unavailability

tu	1	2	3	4
u_{tu}^U (h)	8688	17,448	26,208	34,968
u_{tu}^L (h)	8520	17,280	26,040	34,800

Computational performance of five alternative MILP models involving different combinations of disjunctive constraints and reformulation methods (details given in Table 5) is illustrated through the solution of five test problems of varying difficulty based on the data provided in the previous section. More specifically, we consider the full problem with $|T|=12$ operation periods, $|Tp|=8$ constant electricity price periods, $|Td|=4$ periods of maximum power demand, and $|Tu|=3$ periods where the maintenance team is unavailable; and four subproblems with fewer time periods of operation. The actual data is provided in the first four columns of Table 6, together with key computational statistics.

The main observation from Table 6 is that while all the methods find the same optimal solution, the Hull model is clearly outperformed in terms of CPU time, confirming that the convex hull reformulation of the single maintenance team constraint is not a good option (the hull reformulation of Eqs. 18–22 led to an even worse performance). While the performance of models BM-1, BM-2, Hybrid-1, and Hybrid-2 is quite similar, BM-1 was often the fastest, whereas Hybrid-2 found the best solution and terminated with a lower optimality gap for $|T|=12$. However, BM-2 did return a lower optimality gap than BM-1, due to a lower best possible solution at the time of termination (254.85 vs. 255.72).

The computational statistics in Table 7 help to explain the relative performance of the models. Although sharing the same number of binary variables, the big-M models have the advantage of requiring the fewest variables and constraints but the disadvantage of providing worse linear relaxations (higher integrality gaps). The results for BM-2 show that reorganizing the disjunctions in Eq. 13 into Eqs. 18–22 effectively leads to a smaller and tighter model when compared to BM-1. However, the reduction in integrality gap is considerably smaller than the one that can be achieved with the hull relaxation. Interestingly, the results for Hybrid-1 and Hybrid-2 are the exact opposite, with the former being tighter and requiring fewer constraints (see the discussion in Appendix B). Also notice that the reformulation method chosen for the single maintenance team constraint has a major effect in problem size without affecting the integrality gap. More specifically, model Hybrid-1 requires less than half the number of variables and constraints than Hull and has the same integrality gap. It is thus not surprising that Hybrid-1 achieves better solutions and lower optimality gaps than Hull at the 1-hour termination time (see also Table 8).

It should also be noted in Table 8 that BM-2 provided the best solutions for the five instances with different time periods.

Table 5. Composition of Five MILP Formulations Tested as a Function of Reformulation Strategy used for Each Set of Disjunctions

Model	BM-1	BM-2	Hybrid-1		Hybrid-2		Hull
Disjunctive constraints/reformulation	Big-M	Big-M	Big-M	Hull	Big-M	Hull	Hull
Single maintenance team (Eq. 11)	✓	✓	✓		✓		✓
Unavailability of maintenance team (Eq. 12)	✓	✓		✓		✓	✓
Interaction with electricity price periods (Eq. 13)	✓			✓			✓
Interaction with electricity price periods (Eqs. 18–22)		✓				✓	
Maximum power output (Eq. 16)	✓	✓		✓		✓	✓

Table 6. Key Results for Alternative Reformulations of Disjunctive Programming Model (Best Solutions in Bold, Maximum Computational Time in Italic)

Model				BM-1		BM-2		Hybrid-1		Hybrid-2		Hull	
$\|T\|$	$\|Tp\|$	$\|Td\|$	$\|Tu\|$	Revenue (10^6\$)	CPUs	Revenue (10^6\$)	CPUs	Revenue (10^6\$)	CPUs	Revenue (10^6\$)	CPUs	Revenue (10^6\$)	CPUs
4	3	2	1	**83.64**	0.38	**83.64**	0.51	**83.64**	0.50	**83.64**	0.51	**83.64**	0.66
6	4	2	1	**127.31**	2.28	**127.31**	2.23	**127.31**	3.16	**127.31**	6.68	**127.31**	12.2
8	5	3	2	**163.17**	75.0	**163.17**	356	**163.17**	380	**163.17**	345	**163.17**	3115
10	7	4	3	**213.83**	119	**213.83**	142	**213.85**	279	**213.85**	298	**213.85**	1809
12	8	4	3	251.38	*3600*[a]	250.87	*3600*[b]	251.38	*3600*[c]	**251.45**	*3600*[d]	250.30	*3600*[e]

[a]Optimality gap at termination time (Gap) = 1.73 %.
[b]Gap = 1.59%.
[c]Gap = 1.61%.
[d]Gap = 1.29%.
[e]Gap = 3.35%.

The best found solution for $|T|=12$, representing a revenue of \$$251.448 \cdot 10^6$, is shown in Figure 9. Notice that not all engines are capable of finishing their production tasks within 32,316 h, the upper bound of the last electricity cost period. Twelve engines complete the 12 periods of operation, some long before the end of the horizon, engines M13-M17 complete 11, whereas engine M18 completes just 10. For the overall analysis of the schedule it is, thus, convenient to neglect the terminal effects, roughly after the 27,000-hour mark (three full years of operation).

The most interesting aspect is that there are no idle periods in the high electricity cost periods, as it is desired to meet the goal of maximizing revenue from electricity sales. This can be confirmed by the power output profile that shows a minimum production of 170 MW in the first three green periods, corresponding to a single engine under maintenance. Considering that the labels inside the rectangles give the processing time, a distinction can be made between always-on engines (M1–M9) and the others. Non-always-on engines mostly operate for a time corresponding to the upper bound values given in Table 1, which maximizes productivity. In contrast, always-on engines, predominantly M1, often operate closer to the lower bound to provide enough flexibility to execute all required shutdowns. This point will be discussed further in the next subsection.

The maximum demand periods of 140 MW, in red, are frequently used to perform the mandatory maintenance tasks, as it is apparent in the second of such periods with the 96-hour shutdowns of engines M16–M18. The 432-hour shutdowns, which start around the 17,000-hour mark, are a severe bottleneck on power output. One should, thus, consider hiring additional maintenance crew and incorporating its cost in the objective function. This will be the subject of future work. Finally, while it may appear that there are shutdowns during gray periods, the constraint of no maintenance team available is being satisfied. This is simply due to fact that the picture used as background in the plot area of the chart was generated manually (placing colored rectangles of widths proportional to the duration of the different time periods, one after the other). Although this is good enough to illustrate the main problem features, it is not completely accurate, particularly if the time period is short, as is the case of the no maintenance periods in gray. Bevel shape effects for online tasks also induce in error.

Shutdown schedule flexibility

The mathematical formulation meets the minimum power demand constraint in an indirect way, by not allowing idle times for a subset of the gas engines. Given the single maintenance team constraint, there exists a feasible region only if

Table 7. Computational Statistics Related to Problem Size[a]

Model		BM-1			BM-2			Hybrid-1			Hybrid-2			Hull		
$\|T\|$	BV	TV	E	I.G. (%)	TV	E	I.G. (%)	TV	E	I.G. (%)	TV	E	I.G. (%)	TV	E	I.G. (%)
4	2502	4537	10,641	37	4537	9345	26	8137	13,737	1	8137	14,601	1	17,929	39,081	1
6	5319	9397	22,125	46	9397	19,533	40	16,309	27,849	8	16,309	29,577	9	38,341	84,657	8
8	9540	16,345	38,362	55	16,345	34,042	44	28,297	48,586	6	28,297	51,466	9	67,465	149,386	6
10	16,065	26,461	63,167	50	26,461	55,607	46	47,341	80,897	7	47,341	86,027	11	108,541	239,027	7
12	22,410	36,937	88,115	55	36,937	77,747	51	65,017	111,659	14	65,017	118,571	19	153,145	338,891	14

[a]BV=binary variables; TV=total variables; E=equations; I.G.= integrality gap calculated with respect to best known solution.

Table 8. Computational Statistics Following Increase in Shutdown Flexibility (Best Solutions in Bold, Maximum Computational Time in Italic)

	BM-1				BM-2				Hybrid-1				Hybrid-2				Hull			
$\|T\|$	Revenue (10^6\$)	I.G. (%)	Gap (%)	CPUs	Revenue (10^6\$)	I.G. (%)	Gap (%)	CPUs	Revenue (10^6\$)	I.G. (%)	Gap (%)	CPUs	Revenue (10^6\$)	I.G. (%)	Gap (%)	CPUs	Revenue (10^6\$)	I.G. (%)	Gap (%)	CPUs
4	**83.64**	41	–	0.60	**83.64**	30	–	0.72	**83.64**	3	–	1.00	**83.64**	7	–	1.36	**83.64**	3	–	1.71
6	**127.31**	51	–	1524	**127.31**	47	–	1217	**127.31**	11	–	1377	**127.31**	14	–	2011	**127.31**	11	–	1670
8	**163.64**	58	3.04	*3600*	**163.64**	50	2.32	*3600*	**163.64**	15	1.73	*3600*	**163.64**	22	2.48	*3600*	**163.64**	15	2.16	*3600*
10	214.20	53	4.24	*3600*	**214.21**	50	4.48	*3600*	212.90	19	4.91	*3600*	214.13	26	3.76	*3600*	199.21	19	12.5	*3600*
12	249.55	58	14.5	*3600*	**250.15**	56	12.8	*3600*	249.82	29	10.6	*3600*	244.14	36	16.5	*3600*	No sol.	29	–	*3600*

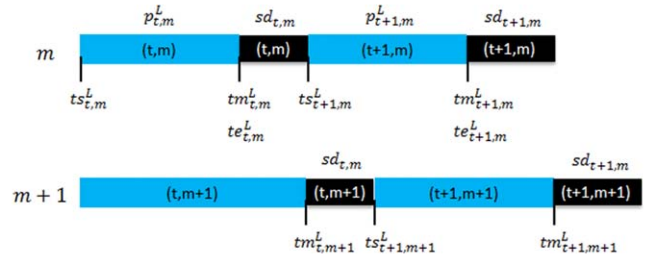

Figure 8. Two possible situations defining the lower bounds on the start of the shutdown tasks.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

the processing times are allowed to vary within a sufficiently wide range. In fact, raising the minimum electricity production to $pw^L=90$ MW makes test problems with $|T| \in \{8, 10, 12\}$ infeasible whereas the remaining two become infeasible for $pw^L=140$ MW. This occurs even after increasing the values of the heuristic upper bounds in Eqs. 80 and 82.

To test the influence of the range of processing times in computational performance, we reduced the values of $p^L_{t,m}$ in Table 1 from 2000 to 1500 h ($t \neq 8$) and 1000 h (t=8). Compared to the base case, the problem is being relaxed, and hence solution quality cannot degrade provided that sufficient time is given to the optimization solver. The results in Table 8 show no improvement in the value of the objective function for $|T| \in \{4, 6\}$, but slightly higher revenues are returned for $|T| \in \{8, 10\}$, (163.64 vs. 163.17 and 214.21 vs. 213.85, compare to Table 6). The drawback is that the computational time increases by at least one order of magnitude, which was difficult to predict (given that the problem size did not change) despite the increase in integrality gap, more pronounced in relative terms for the Hybrid and Hull models. As a consequence, good quality solutions become harder to obtain for the largest problem (notice that for $|T|$= 12 no feasible solution could be found by the Hull model whereas Hybrid-2 returned a rather poor solution).

Overall, the results seem to indicate that reorganizing the disjunctions is better for the big-M reformulation (BM-2), whereas the standard disjunctions are better for the hull reformulation (Hybrid-1). We have also confirmed the well-known capability of big-M formulations to find good solutions in the early nodes of the search tree. Finally, it should be highlighted that optimality gaps around 11% can be too high in the context of revenue in the interval [250, 276] million dollars.

Conclusions

This article has proposed a new continuous-time GDP model for the optimal maintenance scheduling of a gas engine power plant. Emphasis was placed on the derivation of constraints related to the availability of the single maintenance team, a resource shared by the gas engines, to the maximum demand constraints, and to the calculation of the revenue from electricity sales in the given constant-tariff electricity periods. By using the high-level construct of GDP, simple linear constraints could be associated to each decision variable, which were then converted into MILP format using big-M and hull reformulations. In particular, we have shown that the disjunctions linked to revenue

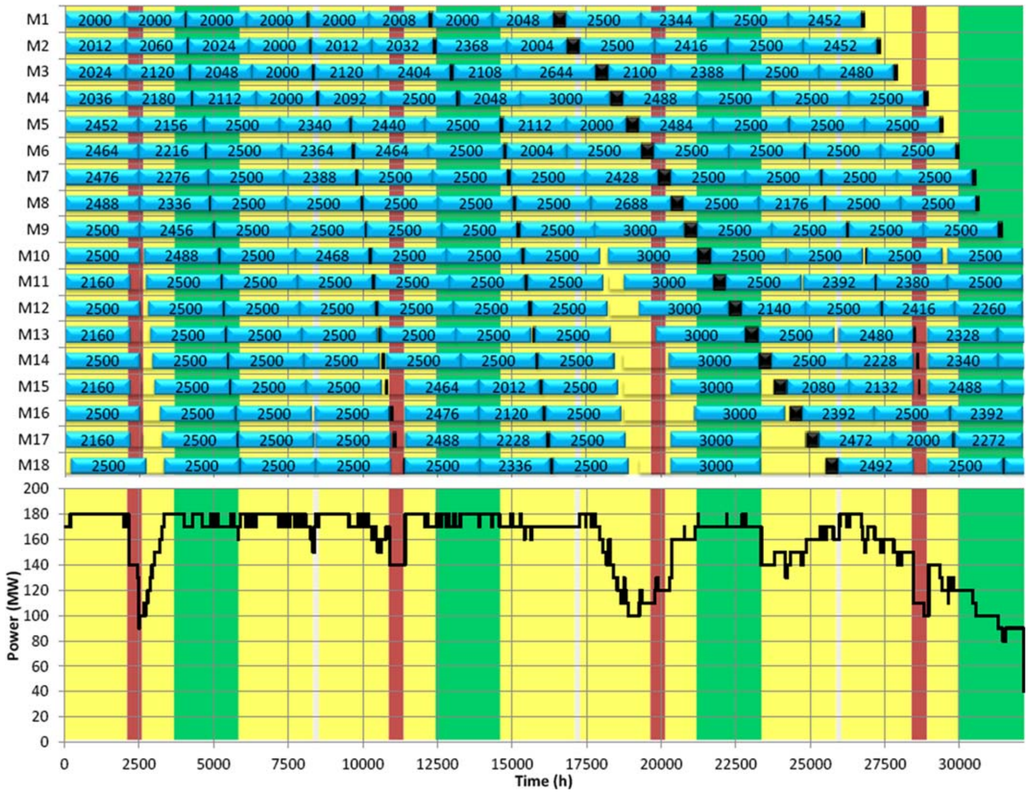

Figure 9. Best found solution for $|T|=12$ operation periods problem (background shows periods with low tariff in yellow, periods with high tariff in green, periods with maximum power demand in red, and periods with no maintenance team available in gray).

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

calculation can be reorganized so that a particular linear constraint appears only once. This has the advantage of leading to smaller and tighter mathematical formulations when using the big-M reformulation, and the disadvantage of leading to larger and looser models when applying the hull reformulation, which is not necessarily translated into a better/worse computational performance. The results have also shown that the hull reformulation of the single maintenance team constraint is particularly inefficient.

Through the solution of an industrial case study featuring identical engines, we have shown that a near optimal (<1.3% optimality gap) maintenance plan can be derived for a time horizon of 3 years, considering seasonal variations in electricity price and other volatile, yet deterministic, resource profiles. Furthermore, we have identified that the single maintenance team becomes an important bottleneck around the time the mandatory shutdowns become longer, significantly reducing power output and revenue. Future work will thus look into the cost-benefit effect of hiring an additional team.

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Appendix A : Convex Hull Reformulation of Eq. 11

We show below the convex hull reformulation of the disjunctions that ensure that the maintenance tasks are not performed simultaneously (Eq. 11). It leads to a major increase in the number of continuous variables and constraints and a significantly worse computational performance than its big-M counterpart.
Let $\widehat{Tm}^{Y}_{t,m,t,m,t',m'}$ and $\widehat{Tm}^{Y}_{t',m',t,m,t',m'}$ be the disaggregated variables associated to the left disjunction, which is reformulated in Eq. A1. Similarly, the right disjunction is reformulated into Eq. A2

$$\widehat{Tm}^{Y}_{t',m',t,m,t',m'} \geq \widehat{Tm}^{Y}_{t,m,t,m,t',m'} + sd_{t,m} \cdot Y_{t,m,t',m'} \;\; \forall \;\; t,t',m' > m \tag{A1}$$

$$\widehat{Tm}^{\neg Y}_{t,m,t,m,t',m'} \geq \widehat{Tm}^{\neg Y}_{t',m',t,m,t',m'} + sd_{t',m'} \cdot (1 - Y_{t,m,t',m'}) \;\; \forall \;\; t,t',m' > m \tag{A2}$$

The original problem variables are related to the new disaggregated variables through Eqs. A3 and A4

$$Tm_{t,m} = \widehat{Tm}^{Y}_{t,m,t,m,t',m'} + \widehat{Tm}^{\neg Y}_{t,m,t,m,t',m'} \;\; \forall \;\; t,t',m' > m \tag{A3}$$

$$Tm_{t',m'} = \widehat{Tm}^{Y}_{t',m',t,m,t',m'} + \widehat{Tm}^{\neg Y}_{t',m',t,m,t',m'} \;\; \forall \;\; t,t',m' > m \tag{A4}$$

Finally, we have the bounding constraints that use the knowledge of the relative position of the two maintenance tasks under consideration (Eqs. A5–A8)

$$tm^{L}_{t,m} \cdot Y_{t,m,t',m'} \leq \widehat{Tm}^{Y}_{t,m,t,m,t',m'} \leq \min\left(tm^{U}_{t,m}, tm^{U}_{t',m'} - sd_{t,m}\right) \cdot Y_{t,m,t',m'} \;\; \forall \;\; t,t',m' > m \tag{A5}$$

$$\max\left(tm^{L}_{t,m}, tm^{L}_{t',m'} + sd_{t',m'}\right) \cdot (1 - Y_{t,m,t',m'}) \leq \widehat{Tm}^{\neg Y}_{t,m,t,m,t',m'} \leq tm^{U}_{t,m} \cdot (1 - Y_{t,m,t',m'}) \;\; \forall \;\; t,t',m' > m \tag{A6}$$

$$\max\left(tm^{L}_{t',m'}, tm^{L}_{t,m} + sd_{t,m}\right) \cdot Y_{t,m,t',m'} \leq \widehat{Tm}^{Y}_{t',m',t,m,t',m'} \leq tm^{U}_{t',m'} \cdot Y_{t,m,t',m'} \;\; \forall \;\; t,t',m' > m \tag{A7}$$

$$tm^{L}_{t',m'} \cdot (1 - Y_{t,m,t',m'}) \leq \widehat{Tm}^{Y}_{t',m',t,m,t',m'} \leq \min\left(tm^{U}_{t',m'}, tm^{U}_{t,m} - sd_{t',m'}\right) \cdot (1 - Y_{t,m,t',m'}) \;\; \forall \;\; t,t',m' > m \tag{A8}$$

Appendix B : Convex Hull Reformulation of Eqs. 18–22

Below, we show the hull reformulation of the disjunctions in Eqs. 18–22, which are essential to compute the revenue due to sales in periods of different electricity pricing. The reformulation involves the same number of disaggregated variables than that for the hull reformulation of Eq. 13, but some of them are different, since instead of being associated to a single interaction with the electricity time period, they are shared by multiple locations. More specifically, Eqs. B1–B3 are the hull reformulation of Eq. 18. Notice that Eq. B1 is not as tight as Eq. 55, whenever $ts^{U}_{t,m} > cp^{U}_{tp}$

$$\max\left[ts^{L}_{t,m}, cp^{L}_{tp}\right] \cdot \left(A_{t,m,tp} + C_{t,m,tp} + F_{t,m,tp}\right) \leq \widehat{Ts}^{\mathrm{ACF}}_{t,m,tp} \leq ts^{U}_{t,m} \cdot \left(A_{t,m,tp} + C_{t,m,tp} + F_{t,m,tp}\right) \;\; \forall \;\; t,m,tp \tag{B1}$$

$$ts^{L}_{t,m} \cdot \left(B_{t,m,tp} + D_{t,m,tp} + E_{t,m,tp}\right) \leq \widehat{Ts}^{\mathrm{BDE}}_{t,m,tp} \leq \min\left[ts^{U}_{t,m}, cp^{L}_{tp}\right] \cdot \left(B_{t,m,tp} + D_{t,m,tp} + E_{t,m,tp}\right) \;\; \forall \;\; t,m,tp \tag{B2}$$

$$Ts_{t,m} = \widehat{Ts}^{\mathrm{ACF}}_{t,m,tp} + \widehat{Ts}^{\mathrm{BDE}}_{t,m,tp} \forall \;\; t,m,tp \tag{B3}$$

Similarly, the reformulation of Eqs. 19–21, leads to the MILP constraints in Eqs. B4–B12

$$ts^{L}_{t,m} \cdot (1 - F_{t,m,tp}) \leq \widehat{Ts}^{\neg F}_{t,m,tp} \leq \min\left[ts^{U}_{t,m}, cp^{U}_{tp}\right] \cdot (1 - F_{t,m,tp}) \;\; \forall \;\; t,m,tp \tag{B4}$$

$$\max\left[ts^L_{t,m}, cp^U_{tp}\right] \cdot F_{t,m,tp} \le \widehat{Ts}^F_{t,m,tp} \le ts^U_{t,m} \cdot F_{t,m,tp} \;\; \forall\; t,m,tp \tag{B5}$$

$$Ts_{t,m} = \widehat{Ts}^{\neg F}_{t,m,tp} + \widehat{Ts}^F_{t,m,tp} \forall\; t,m,tp \tag{B6}$$

$$te^L_{t,m} \cdot E_{t,m,tp} \le \widehat{Te}^E_{t,m,tp} \le \min\left[te^U_{t,m}, cp^L_{tp}\right] \cdot E_{t,m,tp} \;\; \forall\; t,m,tp \tag{B7}$$

$$\max\left[te^L_{t,m}, cp^L_{tp}\right] \cdot (1-E_{t,m,tp}) \le \widehat{Te}^{\neg E}_{t,m,tp} \le te^U_{t,m} \cdot (1-E_{t,m,tp}) \;\; \forall\; t,m,tp \tag{B8}$$

$$Te_{t,m} = \widehat{Te}^E_{t,m,tp} + \widehat{Te}^{\neg E}_{t,m,tp} \;\; \forall\; t,m,tp \tag{B9}$$

$$te^L_{t,m} \cdot (A_{t,m,tp}+B_{t,m,tp}+E_{t,m,tp}) \le \widehat{Te}^{ABE}_{t,m,tp} \le \min\left[te^U_{t,m}, cp^U_{tp}\right] \cdot (A_{t,m,tp}+B_{t,m,tp}+E_{t,m,tp}) \quad \forall\; t,m,tp \tag{B10}$$

$$\max\left[te^L_{t,m}, cp^U_{tp}\right] \cdot (C_{t,m,tp}+D_{t,m,tp}+F_{t,m,tp}) \le \widehat{Te}^{CDF}_{t,m,tp} \le te^U_{t,m} \cdot (C_{t,m,tp}+D_{t,m,tp}+F_{t,m,tp}) \;\; \forall\; t,m,tp \tag{B11}$$

$$Te_{t,m} = \widehat{Te}^{ABE}_{t,m,tp} + \widehat{Te}^{CDF}_{t,m,tp} \;\; \forall\; t,m,tp \tag{B12}$$

The reformulation of Eq. 22 involves Eqs. 65–68 as well as Eqs. B13–B18. Notice that due to the appearance of the $Ts_{t,m}$ and $Te_{t,m}$ variables in Eq. 22, associated to disjunctions $C_{t,m,tp}$ and $B_{t,m,tp}$, we need to define disaggregated variables $\widehat{Ts}^C_{t,m,tp}$, $\widehat{Ts}^{\neg C}_{t,m,tp}$, $\widehat{Te}^B_{t,m,tp}$, and $\widehat{Te}^{\neg B}_{t,m,tp}$. Overall, there are 22 sets of constraints in hull reformulation of Eqs. 18–22, whereas the hull reformulation of Eq. 13, involves just 18

$$\max\left(cp^L_{tp}, te^L_{t,m}\right) \cdot B_{t,m,tp} \le \widehat{Te}^B_{t,m,tp} \le \min\left(cp^U_{tp}, te^U_{t,m}\right) \cdot B_{t,m,tp} \;\; \forall\; t,m,tp \tag{B13}$$

$$te^L_{t,m} \cdot (1-B_{t,m,tp}) \le \widehat{Te}^{\neg B}_{t,m,tp} \le te^U_{t,m} \cdot (1-B_{t,m,tp}) \;\; \forall\; t,m,tp \tag{B14}$$

$$Te_{t,m} = \widehat{Te}^B_{t,m,tp} + \widehat{Te}^{\neg B}_{t,m,tp} \;\; \forall\; t,m,tp \tag{B15}$$

$$\max\left(cp^L_{tp}, ts^L_{t,m}\right) \cdot C_{t,m,tp} \le \widehat{Ts}^C_{t,m,tp} \le \min\left(cp^U_{tp}, ts^U_{t,m}\right) \cdot C_{t,m,tp} \;\; \forall\; t,m,tp \tag{B16}$$

$$ts^L_{t,m} \cdot (1-C_{t,m,tp}) \le \widehat{Ts}^{\neg C}_{t,m,tp} \le ts^U_{t,m} \cdot (1-C_{t,m,tp}) \;\; \forall\; t,m,tp \tag{B17}$$

$$Ts_{t,m} = \widehat{Ts}^C_{t,m,tp} + \widehat{Ts}^{\neg C}_{t,m,tp} \forall\; t,m,tp \tag{B18}$$

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